

# Effective Potentials for Light Moduli

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## Abstract

We examine recent work on compactifications of string theory with fluxes, where effective potentials for light moduli have been derived after integrating out moduli that are assumed to be heavy at the classical level, and then adding non-perturbative (NP) corrections to the superpotential. We find that this two stage procedure is not valid and that the correct potential has additional terms. Although this does not affect the conclusion of Kachru et al (KKLT) that the Kaehler moduli may be stabilized by NP effects, it can affect the detailed physics. In particular it is possible to get metastable dS minima without adding uplifting terms.

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## I. INTRODUCTION

There has been much progress recently in understanding the mechanisms by which the compactification moduli and the dilaton of string theory are stabilized[19]. In particular Giddings et al. [1] discussed type IIB compactification on a Calabi-Yau orientifold  $X$ , and showed that turning on fluxes of NSNS and RR three and five form fields can generate a potential for the complex structure moduli of  $X$  and the dilaton-axion field, which as one would expect is of the supergravity form. However the Kaehler moduli (which includes the overall scale modulus usually denoted by  $T$  in the string-phenomenology literature) cannot be fixed by the fluxes. A suggestion for fixing these was subsequently made by Kachru et al. [2] (KKLT).

The KKLT proposal was to argue that at least for certain choices of fluxes the dilaton-axion ( $S$ ) and the complex structure moduli ( $z^i$ ) would have masses that are close to the string scale and could be integrated out classically to get a theory for the light Kaehler moduli. But at the classical minimum, the potential being of the no-scale type, is zero and does not fix  $T$  (for simplicity we will just consider the case of one Kaehler modulus). In order to get a potential for  $T$  these authors proposed that a contribution coming from certain non-perturbative effects be included. Concretely the argument is that the fluxes which fix  $S$  and  $z^i$  give a constant  $W_0$  in the superpotential, to which the exponential contribution coming from the non-perturbative (NP) effects should be added, resulting in a total superpotential

$$W = W_0 + C e^{-aT},$$

for the theory of the light modulus  $T$ . Here the pre-factor of the NP term is taken to be a constant since we are ignoring perturbative corrections as in KKLT. The Kaehler potential of the theory is taken to be its classical value

$$K = -3 \ln(T + \bar{T}).$$

With this prescription it is easy to see [2] that the modulus  $T$  is fixed at a supersymmetric Anti-deSitter (AdS) point. KKLT go on to lift this minimum by adding a term coming from  $\bar{D}$  branes.

In this note we will examine the consistency of the assumptions that lead to the KKLT theory for the light modulus. These include the expectation that the low energy supergravity

action is a good starting point for finding classical string vacua and also that one can find flux configurations such that the complex structure moduli and the dilaton are heavy. As KKLT observed, in order for the procedure to make sense, the minimum of the potential should be at a large value of  $T$ , so that the size of  $X$  is large on the string scale justifying the ten-dimensional SUGRA starting point, and the superpotential itself is valid only in the region  $aT_R \gg 1$ , so that the NP term can be regarded as a small correction to the classical theory. KKLT argued that although generically the fluxes would give a value of  $W_0$  that is of order one (thus violating the first requirement) there would be (at least for CY manifolds  $X$  with large  $h_{2,1}$ ) flux configurations which would give small values of this constant.

Now of course there are obvious corrections to this theory coming from perturbative effects which, even though they leave the superpotential unchanged, will affect the Kaehler potential. There is also a non-perturbative correction to the Kaehler potential (see for example [3][4]). Thus one would expect a corrected Kaehler potential of the form

$$K = -3 \ln(T + \bar{T} + f + k e^{-a(T+\bar{T})}) \quad (1)$$

where  $f$  is a constant and in principle the coefficient  $k$  could be of the same order as the prefactor  $C$  in the superpotential. We will ignore these corrections in most of this paper and will touch on their effects at the end of our discussion. What we are going to investigate is just the procedure of first ignoring the non-perturbative term in the superpotential in order to integrate out  $S, z^i$  to get a constant superpotential, and then including the non-perturbative term. We will find that if the non-perturbative term is included from the beginning, there are terms which are necessarily controlled by the same coefficient as the terms which are included by KKLT and therefore cannot be set to zero. Related observations have been made by Choi et al. [5] [20] but we will find that there needs to be some modifications of their arguments also. In the course of this investigation we came across some issues in the procedure of integrating out heavy fields in supersymmetric theories that are of general interest, but we will reserve that discussion to a separate publication [6].

The two stage calculation of KKLT appears to lead only to a critical point that is AdS supersymmetric. To get a dS minimum (and broken supersymmetry) KKLT add an uplifting terms namely a contribution from a Dbar brane. An alternate suggestion is to find some sector that gives a D-term [7]. However it is easy to see that such a term will not lift an

AdS supersymmetric minimum. This is because of the relation

$$2\Re f^{ab}D_b = \frac{ik^{ai}D_iW}{W}$$

(with  $k^{ai}$  a generator of a Killing symmetry of the Kahler metric and  $f$  the gauge coupling function) between the D and F terms that is valid at generic points where the superpotential is non-zero [21]. So a critical point where the F-term is zero with  $W \neq 0$  giving an AdS minimum as in KKLT will not be lifted by adding a D term. This also means that a Dbar term as in KKLT if it is to lift the AdS minimum would have to be an explicit breaking term from the point of view of four dimensional supergravity. Of course one can lift a non supersymmetric ( $D_iW \neq 0$ ) AdS critical point (such an example can be found in [8]) by a D-term.

One of the outcomes of the current investigation is that if one integrates out the heavy moduli in one stage then one has extra terms in the potential (compared to the two stage procedure). These terms enable one to find examples where the F-term potential by itself has positive local minima thus obviating the need for uplifting terms.[22] In view of this it would be interesting to revisit other issues such as the question of getting a viable cosmology in the context of such models.

## II. MODEL WITH S AND T

Consider first a compactification with fluxes on a rigid CY manifold  $X$ , i.e. one with  $h_{21} = 0$ .

The classical Kaehler potential is

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}), \quad (2)$$

and with the non-perturbative contribution included we have for the superpotential,

$$W = A + SB + Ce^{-aT}, \quad (3)$$

where  $A, B$  are determined by the fluxes and  $C$  is an  $O(1)$  prefactor which may be determined by an instanton calculation. Now let us first solve for  $S$  in terms of  $T$  as in KKLT by requiring that the Kaehler derivative with respect to  $S$  of this superpotential is zero [23]. This gives,

$$D_SW = B - \frac{A + SB + Ce^{-aT}}{S + \bar{S}} = 0, \quad (4)$$

implying

$$\bar{S} = (A + Ce^{-aT})/B. \quad (5)$$

Clearly if we substitute this back into (3) we get an expression which contains both  $T$  and  $\bar{T}$  i.e. it is not holomorphic. As far as the scalar potential goes this is not a problem - the coupling of the chiral scalars to supergravity is actually determined by one Kaehler invariant real function (see for example [9] or [10])

$$G = K(\Phi, \bar{\Phi}) + \ln W(\Phi) + \ln \bar{W}(\bar{\Phi}). \quad (6)$$

After solving for  $S$  this becomes

$$\begin{aligned} G = & -\ln\left(\frac{(A + Ce^{-aT})}{B} + \frac{(\bar{A} + \bar{C}e^{-a\bar{T}})}{\bar{B}}\right) - 3\ln(T + \bar{T}) \\ & + \ln\left(A + B\frac{(\bar{A} + \bar{C}e^{-a\bar{T}})}{\bar{B}} + Ce^{-aT}\right) + c.c.). \end{aligned} \quad (7)$$

This may in effect be regarded as the new Kaehler potential with the superpotential being taken to be unity. The potential for  $T$  may now be computed from the standard formula

$$V = e^G (G_i G_{\bar{j}} G^{i\bar{j}} - 3), \quad (8)$$

where  $G_i = \partial G / \partial \Phi^i$  and  $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} G$  is the Kaehler metric.

On the other hand if we had followed the prescription of KKLT we would have solved for  $S$  in the absence of the non-perturbative term to get  $\bar{S} = A/B$ , a constant superpotential  $W = A + B\bar{A}/\bar{B} \equiv W_0$  and apart from an irrelevant constant the Kaehler potential is  $K = -3\ln(T + \bar{T})$ . Now the non-perturbative term is added to  $W$  to get a Kaehler invariant function,

$$G = -3\ln(T + \bar{T}) + (\ln(A + B\frac{\bar{A}}{\bar{B}} + Ce^{-aT}) + c.c.) \quad (9)$$

The problem is that in this two stage process one is ignoring non-perturbative terms in (7) that are in fact controlled by the same constant  $C$  as the terms that are being kept. *There is no approximation in which one can keep the latter and ignore the former.* In other words the procedure of first integrating out  $S$  and then adding the non-perturbative term to  $W$  cannot be justified. It should be noted also that this correction is of the same order as the term kept by KKLT independently of the condition  $W_0 \ll 1$  required by KKLT.

### III. MODELS WITH COMPLEX STRUCTURE MODULI

The model studied in the previous section however does not give a viable theory in any case. Choi et al [5] have analyzed the stability of this model (without first integrating out the dilaton-axion  $S$ ). They find that the supersymmetric extremum  $D_S W = D_T W = 0$  is in fact a saddle point. Although this does not make the supersymmetric point unstable (since a saddle point or even a maximum can be a stable AdS solution) it becomes problematic when one adds a “lifting potential” as in KKLT to get a dS solution, since it is unlikely that such a corrected potential would have a stable critical point and indeed that is what Choi et al find. As they have argued, the point is that the mass of the field that is integrated out depends on the light field and thus it cannot be integrated out as suggested by KKLT. Our argument above highlights this point directly by showing that the procedure of KKLT ignores effects that simply cannot be set to zero or assumed to be small.

Choi et al go on to analyze models with complex structure moduli. However the analysis is done by assuming that the complex structure moduli can be integrated out holomorphically (resulting in a holomorphic superpotential) to get a potential in just  $S$  and  $T$ . We will find that this procedure is not consistent and has the same problems that we highlighted before.

The Kaehler potential is now

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + k(z^i, \bar{z}^{\bar{j}}). \quad (10)$$

Here  $k = -\ln \int \Omega \wedge \bar{\Omega}$  (with  $\Omega$  being the holomorphic 3-form on the Calabi-Yau space) is the Kaehler potential on the complex structure moduli space (with complex coordinates  $z^i, i = 1, \dots, h_{21}$ ). Also we have assumed that there is only one Kaehler structure. The superpotential is taken to be

$$W = A(z^i) + SB(z^i) + Ce^{-aT}. \quad (11)$$

The Kaehler derivatives with respect to the chiral scalars are,

$$D_T W = -aCe^{-aT} - \frac{3}{T + \bar{T}} W, \quad (12)$$

$$D_S W = B - \frac{W}{S + \bar{S}}, \quad (13)$$

$$D_i W = \partial_i A + S \partial_i B + \partial_i k W. \quad (14)$$

Thus there are  $h_{12} + 2$  complex equations for as many complex variables ( $h_{12}$  complex structure moduli, one Kaehler modulus and the dilaton-axion) so that all of them can be

fixed. Choi et al assume that the equation  $D_i W = 0$  can be solved holomorphically, giving [24] an effective theory for  $S$  and  $T$  with a superpotential

$$W = W_{eff} + C e^{-aT}$$

where

$$W_{eff} = A(z^i(S, T)) + B(z^i(S, T))S.$$

and a Kaehler potential

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + k(z(S, T), \bar{z}(\bar{S}, \bar{T})) \quad (15)$$

The SUSY conditions in the effective theory are,

$$\begin{aligned} D_S W &= D_S W|_{z^i} + \frac{\partial z^i}{\partial S} D_i W = 0 \\ F_T &= D_T W|_{z^i} + \frac{\partial z^i}{\partial T} D_i W = 0 \end{aligned}$$

which are of course implied by the equations of the original theory  $D_S W = D_T W = D_i W = 0$ , with the chiral fields being all independent variables. However this equivalence is guaranteed only if we do not ignore the last term in (15). In Choi et al however the effective Kaehler potential is taken to be just the first two terms of (15). This is not really consistent and it is in fact the dependence of  $k$  on  $z^i$  as well as  $\bar{z}^{\bar{i}}$  that makes it impossible to find an holomorphic solution for  $S$  and  $T$  in terms of  $z^i$  and hence a holomorphic  $W_{eff}$ . To see this consider the equation that needs to be solved,

$$D_i W = \partial_i A(z^i) + S \partial_i B(z^i) + W(S, T, z^i) k_i = 0. \quad (16)$$

This is supposed to have solutions  $z^i = z^i(S, T)$  such that  $\partial_{\bar{S}} z^i = \partial_{\bar{T}} z^i = 0$  for some range of values of  $S$  and  $T$ . So differentiating (16) with respect to  $\bar{S}$  we get from the assumed holomorphicity,

$$W(S, T, z^i(S, T)) k_{i, \bar{j}} \frac{\partial \bar{z}^{\bar{j}}}{\partial \bar{S}} = 0.$$

But the superpotential should not vanish (except at particular points) and  $k_{i, \bar{j}}$  is the Kaehler metric on the complex structure moduli space which is non-degenerate. Hence the above equation implies  $\frac{\partial z^i}{\partial \bar{S}} = 0$  and similarly  $\frac{\partial z^i}{\partial \bar{T}} = 0$ ! Clearly what is at fault is the assumption of holomorphicity. In other words the solution of (16) must be of the form  $z^i = z^i(S, T, \bar{S}, \bar{T})$ .

As we saw explicitly in the case without complex structure moduli where  $S$  was integrated out, in supergravity, fields cannot be integrated out in a holomorphic fashion. As in that case, we expect that the effective supergravity theory is one with a superpotential that is unity and a Kaehler potential

$$G = K = -\ln(S+\bar{S})-3\ln(T+\bar{T})+k(z(S,T,\bar{S},\bar{T}),\bar{z}(\bar{S},\bar{T},S,T))+\ln|W(S,T,z(S,T,\bar{S},\bar{T}))|^2.$$

Clearly similar remarks would apply to an effective theory that is obtained by integrating out both the complex structure moduli as well as the dilaton-axion to get an effective theory for  $T$ .

#### IV. EFFECTIVE POTENTIAL FOR T

Let us now try to find the effective potential for the modulus  $T$  (assumed light) after integrating out the complex structure moduli  $z_i$  and the dilaton-axion  $S$  which we assume to be heavy. Such an effective potential is useful for cosmological applications. Hitherto it has been derived using the two stage process of KKLT, but as we have already seen in section 3 even in the absence of the  $z_i$  there are terms in the effective potential that are as large as the terms that are kept in the two stage argument but were ignored there.

The potential below the string and Kaluza-Klein scale is of the standard  $N = 1$  SUGRA form with the Kaehler and superpotentials being given respectively by eqns (10) (11). To classically integrate out the  $z_i$  and  $S$  we need to solve eqns (12) (14) for these variables in terms of  $T$  and then plug those solutions into the expression for the potential given in (8) and (6). However the equations to be solved are non-linear in the  $z_i$  so the best we can do is to write the general form of the solution in a power series expansion in  $Ce^{-aT}$  for  $aT \gg 1$ . So we write,

$$\begin{aligned} S &= \alpha + \beta Ce^{-aT} + \gamma \bar{C} e^{-a\bar{T}} + \dots \\ z^i &= \alpha^i + \beta^i Ce^{-aT} + \gamma^i \bar{C} e^{-a\bar{T}} + \dots \end{aligned} \tag{17}$$

where the ellipses denote higher order terms in  $Ce^{-aT}$ . The coefficients  $\alpha, \beta, \gamma$  are functions of the (integer) fluxes. To compare with the KKLT two stage calculation we actually need to keep terms up to second order. If we plug this expansion into the expression for  $G$  we get



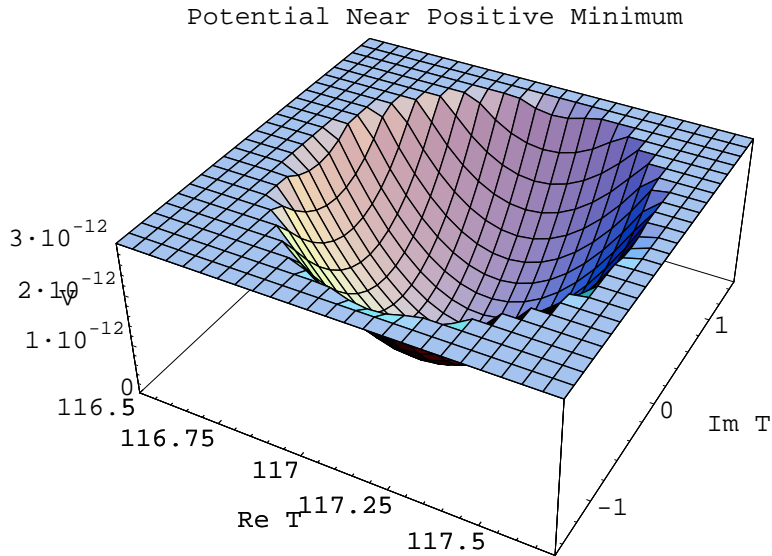
$$G = \ln(v + bCe^{-aT} + \bar{b}\bar{C}e^{-a\bar{T}} + cC^2e^{-2aT} + \bar{c}\bar{C}^2e^{-2a\bar{T}} + d|C|^2e^{-a(T+\bar{T})} + \dots) - 3\ln(T + \bar{T}) \quad (18)$$

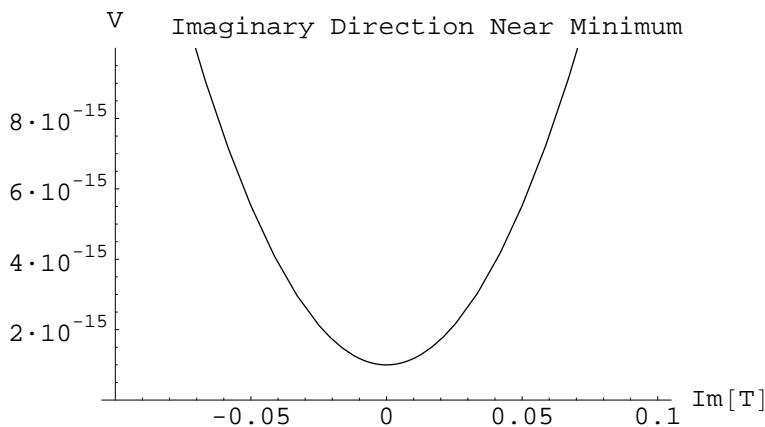
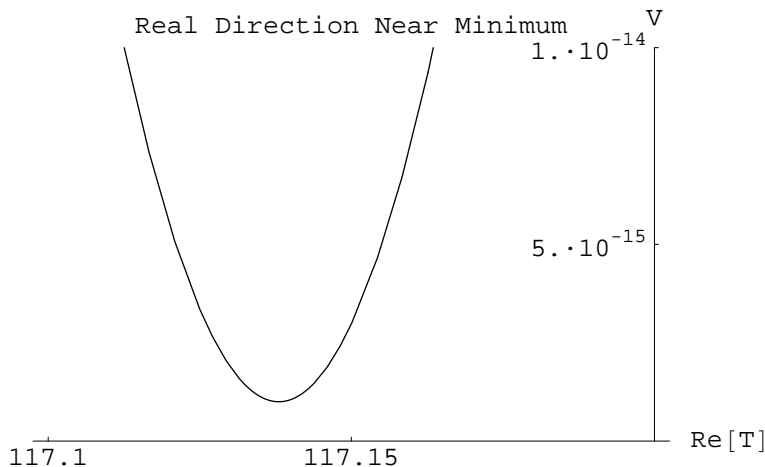
with the new constants (note that  $v, d$  are real) being functions of the ones in (17) and hence of the flux integers. Calculating the potential using (8) then gives

$$V = \frac{1}{(T + \bar{T})^2} [a(bCe^{-aT} + 2cC^2e^{-2aT} + c.c.) + a|C|^2((4\frac{a|b|^2}{v} - 3ad)\frac{T + \bar{T}}{3} + 2d)e^{-a(T+\bar{T})}] \quad (19)$$

Note that the terms  $cC^2e^{-2aT} + c.c.$  would not have been present if we had done the calculation in two stages as in [2]. The expression is also different even in the real direction of the potential since now we have more parameters. Since the functional dependence of the parameters in the potential on the flux integers is hard to evaluate explicitly and in any case is model dependent, we believe that the only real test of the implications of the potential coming from type IIB flux compactifications is to confront this general form of  $V$  with experiment/observation.

In fact one of the immediate consequences of the above form of the potential is that it is possible to find metastable deSitter minima. A simple example (with just one condensate) is illustrated in the figures below. This example has the following parameters for (19):  $a = \frac{2\pi}{320}$ ,  $v = 0.22941751641574312$ ,  $b = 1$ ,  $c = -1.4097828718993035$ ,  $d = 15.786002156414208$ , and  $C = 1$ . The minimum is at  $\Re T_{min} = 117.138$ ,  $\Im T = 0$ ,  $V_{min} = 10^{-15}$  with  $M_p = 1$ .





Such potentials may also have a supersymmetric AdS minimum at  $G_T = 0$  though in this particular example this seems to be absent. Also as is the case for all moduli potentials in string theory, for large  $\Re T$  the potential goes to zero and the positive minimum is only meta-stable.

A comment about the parameters chosen for our example is in order here. The parameters are chosen to give a positive minimum at a reasonable large value of ( $\Re T \sim 10^2$ ) such that also the exponential factor  $e^{-a\Re T} \sim 0.1$  is small thus justifying the perturbation expansion of adding instanton corrections to the classical superpotential. Of course to get a critical point from such an expansion obviously two or more of such an expansion have to be of the same order. Thus here the term  $de^{-a(T+\bar{T})} \sim 0.16$  is of the same order as constant  $v \sim .2$  and  $be^{-a\Re T} \sim 0.1$ . This doesn't necessarily mean that the perturbation expansion is violated. It just means that some of these coefficients need to be fine tuned in order to get a critical point though the generic coefficients would be expected to be of  $O(1)$ . This is inevitable

in any such calculation of the KKLT type (in the original calculation  $W_0 \sim Ce^{-aT}$  at the critical point and is anomalously small) since the existence of a critical point requires that the classical terms be balanced by the non-perturbative correction terms. For perturbation theory to be violated one would need the coefficients of the higher order terms to continue to grow like  $e^{a\Re T}$  and this is extremely unlikely.[25]

So far we have worked with just one condensate. If we have several (so that  $W \sim \sum C_i e^{-a_i T}$ ) then one would need to make the following replacements

$$\begin{aligned}
abCe^{-aT} &\rightarrow \sum_i a_i b_i C_i e^{-a_i T} \\
acC^2 e^{-2aT} &\rightarrow \sum_{ij} (a_i + a_j) c_{ij} C_i C_j e^{-(a_i + a_j)T} \\
a^2 |b|^2 |C|^2 e^{-a(T+\bar{T})} &\rightarrow \sum_{ij} a_i a_j b_i \bar{b}_j C_i \bar{C}_j e^{-(a_i T + a_j \bar{T})} \\
a^2 d |C|^2 e^{-a(T+\bar{T})} &\rightarrow \sum_{ij} a_i a_j d_{ij} C_i \bar{C}_j e^{-(a_i T + a_j \bar{T})} \\
2ad |C|^2 e^{-a(T+\bar{T})} &\rightarrow \sum_{ij} (a_i + a_j) d_{ij} C_i \bar{C}_j e^{-(a_i T + a_j \bar{T})}
\end{aligned}$$

Finally we note that if one includes the leading perturbative correction and non-perturbative corrections to the Kaehler potential then one would need to replace  $-3 \ln(T + \bar{T}) \rightarrow -3 \ln(T + \bar{T} + f + ke^{-a(T+\bar{T})})$  with  $f, k$  constants. Such an addition will clearly not change the qualitative features of the potential.

## V. CONCLUSIONS

In this note we examined the validity of the KKLT procedure of first classically integrating out the dilaton-axion and the complex structure moduli, to obtain an effective theory for the Kaehler moduli, and then adding a non-perturbative term to the superpotential to obtain a potential that stabilizes the Kaehler moduli. We find that there is no approximation scheme in which the procedure of first integrating out the  $S$  and  $z^i$  fields classically and then adding a  $T$ -dependent non-perturbative term to the superpotential is justified. The latter term needs to be included from the beginning and gives additional contributions to the potential that cannot be ignored. Also we find that the procedure cannot be done holomorphically, i.e. the effective theory has to be defined entirely in terms of a Kaehler potential (in effect the Kaehler invariant function  $G$ ) and a superpotential that is just unity. These considerations of course do not affect the result of KKLT that the Kaehler

modulus can be stabilized by non-perturbative effects. But it does change the form of the potential for the Kaehler modulus so that the physical effects (in particular the cosmological considerations) emerging from the theory need to be reconsidered. For instance in [8] it was shown that if one follows the KKLT procedure, then there is no way of getting a broken supersymmetric minimum with a positive or zero cosmological constant with just one light modulus  $T$ , (even with an arbitrary number of non-perturbative terms) without adding an uplifting term. However this is no longer the case if one correctly integrates out the heavy moduli, and we showed in an explicit example that it is possible to obtain a positive local minimum with just the F-term potential. Also the cosmological considerations based on KKLT such as [11][12], need to be revisited in light of the present results. In addition to the effects considered here, perturbative corrections to the Kaehler potential also need to be taken into account. A complete treatment of the physics of such models, the possibility of getting small supersymmetry breaking, a small cosmological constant, sufficient inflation etc. after including these corrections, will be discussed in forthcoming work [26].

## VI. ACKNOWLEDGMENTS

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- [1] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. **D66**, 106006 (2002), hep-th/0105097.
  - [2] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi (2003), hep-th/0301240.
  - [3] V. Kaplunovsky and J. Louis, Nucl. Phys. **B422**, 57 (1994), hep-th/9402005.
  - [4] C. P. Burgess, J. P. Derendinger, F. Quevedo, and M. Quiros, Annals Phys. **250**, 193 (1996), hep-th/9505171.
  - [5] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, JHEP **11**, 076 (2004), hep-th/0411066.
  - [6] S. P. de Alwis (2005), hep-th/0506267.
  - [7] C. P. Burgess, R. Kallosh, and F. Quevedo, JHEP **10**, 056 (2003), hep-th/0309187.

- [8] R. Brustein and S. P. de Alwis, Phys. Rev. **D69**, 126006 (2004), hep-th/0402088.
- [9] J. Wess and J. Bagger, Supersymmetry and supergravity (1992), princeton, USA: Univ. Pr. 259 p.
- [10] S. J. Gates, M. T. Grisaru, M. Rocek, and W. Siegel, Superspace, or one thousand and one lessons in supersymmetry - Front. Phys. **58**, 1 (1983), hep-th/0108200.
- [11] S. Kachru et al., JCAP **0310**, 013 (2003), hep-th/0308055.
- [12] J. J. Blanco-Pillado et al., JHEP **11**, 063 (2004), hep-th/0406230.
- [13] V. Balasubramanian, Class. Quant. Grav. **21**, S1337 (2004), hep-th/0404075.
- [14] E. Silverstein, TASI / PiTP / ISS lectures on moduli and microphysics (2004), hep-th/0405068.
- [15] D. Lust, S. Reffert, W. Schulgin, and S. Stieberger (2005), hep-th/0506090.
- [16] G. Curio, A. Krause, and D. Lust (2005), hep-th/0502168.
- [17] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, Nucl. Phys. **B718**, 113 (2005), hep-th/0503216.
- [18] V. Balasubramanian and P. Berglund, JHEP **11**, 085 (2004), hep-th/0408054.
- [19] For recent reviews with references to the original literature see [13] [14].
- [20] See also [15]. Related issues in the heterotic context have been discussed in [16].
- [21] This relation can be found for example in [10] eqns. (8.7.7b) (8.7.8). It has been recently rediscovered in the current context in [17]. The right hand side of this equation can also be rewritten as  $ik^{ai}\partial_i K + \xi \text{tr} T^a$  (with  $\xi$  a FI parameter) and would give an independent condition if  $W = 0$ .
- [22] It is possible that  $\alpha'$  corrections also achieve the same end. See for instance [18].
- [23] There are some issues involved in integrating out heavy fields in supersymmetric theories that have not been discussed in the literature. In particular it turns out that even in global supersymmetric theories the condition  $\partial_H W = 0$  which is imposed in order to integrate out a heavy field  $H$  is valid only if we also restrict the light field space to range over values that are less than the mass of the heavy field. A similar restriction holds in supergravity. These issues are discussed in a recent paper by the author [6]. In particular it is shown there that solving the Kaehler derivative equated to zero for the heavy field is an acceptable method of computing the bosonic effective potential for the light fields even though to get the complete action for the light fields additional terms involving the fermions need to be kept.
- [24] Choi et al ignore the  $T$  dependence but we keep it here, in any case the relevant issue is

holomorphy.

- [25] This is reminiscent of a well known argument in large N gauge theory - the so-called Banks-Zaks fixed point - which is obtained by cancelling different orders in perturbation theory, but justified on the grounds that the resulting fixed point is still at small coupling. The argument being that one of the coefficients of the perturbation expansion is anomalously large thus giving the cancellation required to get a fixed point, but that the higher order terms could still be expected to have coefficients that did not grow with the power of the coupling constant.
- [26] R. Brustein, S.P. de Alwis and P. Martens - work in progress.